

Title: Nonlocal (fractional) problems and their mathematical models: Numerical analysis and computation

Abstract:

Nonlocal interactions frequently arise in nature and have found applications across various scientific and engineering domains. This work presents a comprehensive study on the construction, analysis, and implementation of advanced numerical methods for solving a wide range of nonlocal and fractional partial differential equations (PDEs). Special emphasis is placed on energy-preserving and structure-preserving techniques, particularly for systems involving Riesz and Caputo fractional derivatives. Both classical and emerging models from physics, biology, and finance are considered, with traditional models extended to fractional and variable-order frameworks. A variety of high-order finite difference and spectral Galerkin methods are developed and rigorously analyzed with respect to stability, convergence, and consistency. Key contributions include the formulation of robust numerical schemes for nonlinear wave equations, reaction-diffusion systems, fractional Schrödinger equations, and delay differential equations-many of which exhibit dissipative or conservative behavior. Theoretical analysis and numerical experiments demonstrate that the proposed methods are accurate, efficient, and adaptable, offering promising directions for future research in fractional dynamics and computational mathematics. In summary, this work develops and analyzes a broad suite of numerical methods tailored for fractional and nonlocal PDEs, contributing novel schemes and theoretical insights.